Lecture 33

In this lecture we look at some more examples of binomial probabilities.

Example

Volunteers answering questions in a numeracy experiment have probability 80% of having the "zero parity confusion factor" whereby they don't know whether zero is even or odd.

(a) If 5 are selected, what's the probability that at least one doesn't have the zero parity confusion factor?

(b) If 5 are selected, what's the probability that at most four of them do have the zero parity confusion factor?

Solution

Let p = the probability of "success"

= Pr(having the zpc factor)

= 80% = 0.8.

(a)

Then Pr(at least 1 doesn't have zpc)

$= 1 - \Pr(\text{all 5 have zpc})$

$= 1 - {}^5C_5 (0.8)^5 (0.2)^0$

$= 1 - 1 \cdot (0.32768) \cdot 1$

$= 0.67232.$

Alternatively,

Pr(1, 2, 3, 4 or 5 failures)

= Pr(4, 3, 2, 1 or 0 successes)

$= \Pr(X \leq 4)$ (where X = no. of successes)

= 0.672 (from the Tables).

(b) Pr(at most 4 successes) $= \Pr(X \leq 4)$

= 0.67232 (as in (a) above).

Example

The irrational fear of mathematics is known as maths phobia. The probability that a patient recovers from maths phobia is 0.8. Suppose 20 people are known to have contracted the phobia.

(a) What is the probability that exactly 14 people recover?

(b) What is the probability that at least 10 recover?

(c) What is the probability that between 14 and 18 recover?

(d) What is the probability that at most 16 recover?

Solution

(a) $Pr(X=14) = {}^{20}C_{14}(.8)^{14}(.2)^{6}$

$= 0.1091$ (using a calculator)

Or: $Pr(X=14) = Pr(X \leq 14) - Pr(X \leq 13)$

$= .196 - .087$ (Tables)

$= .109$

(b) $Pr(X \geq 10) = 1 - Pr(X \leq 9)$

$= 1 - 0.001$ (Tables)

$= 0.999$

(c) $Pr(14 \leq X \leq 18) = Pr(X \leq 18) - Pr(X \leq 13)$

$= .931 - .087$ (Tables)

$= .844$

(d) $Pr(X \leq 16) = .589$ (Tables)